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## Filtering Aspects of Orbit Determination

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OCTOBER 1965

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*Prepared for* COMMANDER SPACE SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
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EL SEGUNDO TECHNICAL OPERATIONS • AEROSPACE CORPORATION  
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**FILTERING ASPECTS OF ORBIT DETERMINATION**

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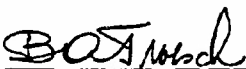
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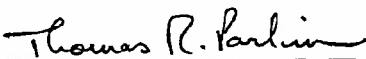
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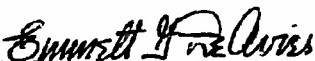
  
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## ABSTRACT

An exciting prospect for modern filtering theory lies in the field of orbital navigation, but as of today the most impressive results in orbit determination have been obtained by classical methods of parameter estimation. Some current experience and problems are described and their implications for modern filtering theory are discussed. Two techniques, found effective in the classical process, are presented in the context of recursive estimation. Four illustrative plots are included.

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## 1. INTRODUCTION

Many early expositions of the modern filtering theory (MFT) have been in orbit determination and some of the most exciting prospects for the implementation of MFT lie in the field of space navigation (Refs. 1 and 2). However, as of today, the most impressive results in orbit determination have been obtained with the classical methods of parameter estimation, as applied at the Naval Weapons Laboratory, Applied Physics Laboratory (APL) of The Johns Hopkins University, and at other installations (Refs. 3, 4, and 5). Some of the experience and problems of the classical approach will be described and their implications for MFT discussed. Two techniques used effectively in the classical process are presented in the context of recursive estimation.

## 2. THE PROBLEMS

The typical problems of MFT (smoothing, filtering, and prediction) as defined by Kalman (Ref. 6) correspond in the field of orbit determination to those of ephemeris reconstruction, navigation, and orbital prediction, all of which have important applications. The navigational or filtering problem (that of seeking a current estimate of position and velocity based on a current observation and a recent past estimate) is one that astronauts will face with the advent of onboard navigation, but it has not commanded much specific attention in classical orbit determination. As for the smoothing and prediction problems, the ephemeris of a geodetic satellite must be reconstructed with all possible precision, and a navigational satellite is useful only if its position can be predicted with great accuracy. There have been impressive results in both areas.



### 3. THE RECORD

In both problems described, primary concern is with the satellite ephemeris, that is, with its position and velocity as functions of time. A variety of empirical methods have been used to increase the accuracy of the ephemeris. Nearly all were found wanting and replaced, with improved results, by more sophisticated estimation procedures.

Following are two related examples: In some early attempts at orbit reconstruction at Aerospace Corp., the final residuals from several flights were much too large (several thousands of feet) for the quality of the data, but could be accounted for very nicely with a time correction. Timing error curves were prepared and used as an empirical device for increasing the accuracy of the ephemeris. Their shape (sinusoidal with a 12-hour period) strongly suggested that the principal longitude-dependent term (the  $J_{22}$  term) should be included in the geopotential. This was done, and a suitable value for  $J_{22}$  did indeed effect a dramatic reduction in the residuals. The acid test of a scientific hypothesis being prediction, the revised geopotential, if a true scientific advance and not just a fancier empirical device, would have to lead to smaller residuals on other flights as well. It did, somewhat, but for best results required a different value for  $J_{22}$ , again due to further deficiencies in the geopotential model. We were soon in the business of estimating  $J_{22}$  and a few other coefficients in the geopotential, as well as a drag coefficient, in addition to the six orbit parameters. All this was in quest of just a position and velocity ephemeris.

Additional degrees of freedom will invariably reduce the residuals in the fitted interval, but this is no proof that a better model has been found. As mentioned, the proper test of a hypothesis is in prediction; and the use of empirical techniques to obtain a good reconstruction has invariably resulted in rapidly increasing residuals outside the fitted interval. Ultimately, only the true model will pass the test of prediction.

The outstanding example of precision orbit determination today is, possibly, the work on the TRANSIT project by APL. The project requires the prediction of a precision position and velocity ephemeris, and thus a highly accurate knowledge of the geopotential. (Their satellites are sufficiently high so that drag, although it cannot be ignored, is not a problem.) To fulfill their mission, they have had the huge problem of estimating 70 non-zonal parameters of the geopotential with 1662 passes of satellite data (from 5 satellites) taken at uncertain locations (Refs. 4 and 5).

Working with the same data, but with independent computer programs, the Naval Weapons Laboratory has also made an impressive determination of the coefficients of the geopotential (Ref. 3). In their program, as many as 500 parameters are estimated simultaneously.

Inspection of residuals played an important role in their work, as it has in ours. Noting a pronounced oscillation of period about 60 hours, APL deduced a resonance effect between the orbital period of the satellite and non-zonal harmonics in the geopotential of order 13, with a beat period of 60 hours (Ref. 7). They solved for the appropriate geopotential coefficients, and noted an appropriate reduction in residuals on two satellites launched three months apart (the prediction test), thereby completing an iteration of the hypothesis-solution-test cycle.

The remaining residuals exhibit a daily pattern correlated with the longitude of the ascending node and, for some satellites, with the latitude. Only 10 to 30 m of the remaining residuals (less than 80 m) appear to be essentially random.

These achievements are very impressive. Not only are good fits obtained in reconstruction, but also their results satisfy several important and independent checks, and pass the test of prediction brilliantly.

#### 4. ESTIMATING UNCERTAINTIES

There are two types of measures of the uncertainty in a reconstructed ephemeris. One (which might be called "practical") is derived from an examination of the residuals; the other, the covariance matrix, is (under certain conditions) the inverse of the normal matrix  $A'WA$ ,<sup>\*</sup> and is thus a theoretical measure in which the residuals have no part.

It is easy to stage examples wherein small changes in the geophysical model can have large effects upon the residuals, but virtually none upon the covariance matrix. These do not speak well for the value of the covariance matrix, as a measure of uncertainty. The explanation is that the conditions under which the covariance matrix is valid are overlooked - as usual. The conditions are that the random observational errors have mean zero and covariance matrix  $W^{-1}$  and that the model be correct.

The latter requirement is invariably violated, in orbit determination problems, by linearization of the original equations, and more subtly, every time a geopotential or atmospheric model is used; for, inevitably, neither the true forms of the models nor the coefficients involved are certainly known. Nor are the locations of the observing stations or the systematic error characteristics of the observing instruments precisely known. Thus the usual covariance matrix determined in the course of estimating six (or a few more) orbit parameters, assuming all other parameters are exactly known, may be (depending on the sophistication and accuracy of the model) hopelessly optimistic and virtually worthless as a measure of ephemeris accuracy.

Yet it is precisely this matrix that governs the size of corrections to MFT state variables. A small covariance matrix, implying a high but false confidence in the current estimates, will permit only small corrections thereto. Clearly, some techniques that tend to de-weight old observations will be required.

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<sup>\*</sup>The prime denotes the transpose of a vector or matrix.

For system design studies, however, there is no recourse to uncertainty estimates derived from observations. Two alternatives are a Monte Carlo analysis (of frightening dimensions for a typical orbit determination problem), or an expanded form of covariance analysis described in Refs. 8, 9, and 10 and below.

It is assumed here that the form of a linear model is known, and that certain parameters or state variables are to be estimated, but in the presence of other parameters the values of which are uncertain and not being estimated. From the first assumption, it is apparent that the method has some deficiencies, but may permit sensible answers to relative questions such as: "How does the uncertainty in the range bias of Station A, which is not being estimated, degrade our positional accuracy?"

Suppose that observational residuals can be represented, after linearization, as the vector

$$y = Ax + Bv + \epsilon \quad (1)$$

where

- $y$  is an  $n$ -vector of residuals
- $x$  is a  $p$ -vector of parameters to be estimated
- $v$  is the  $q$ -vector of parameters not being estimated but with uncertainties given by  $E(vv') = P_v$
- $A, B$  are matrices ( $n \times p$  and  $n \times q$ , respectively)
- $\epsilon$  is an  $n$ -vector of random observational errors, of zero mean and covariance matrix  $E(\epsilon\epsilon') = W^{-1}$

In deriving the estimate  $\hat{x}$  of  $x$ , however, the dependence upon  $v$  is ignored. Thus the estimate is the familiar

$$\hat{x} = (A'WA)^{-1}A'Wy \quad (2)$$

which is readily seen to be an unbiased estimate of  $x$  if  $E(v) = 0$  (meaning that unbiased estimates were used as nominal values of the unknown parameters). Then

$$\delta x = \hat{x} - x = (A'WA)^{-1}A'W(Bv + \epsilon) \quad (3)$$

and the covariance matrix of the estimate is, assuming  $E(v\epsilon') = 0$ ,

$$E(\delta x \delta x') = (A'WA)^{-1} + (A'WA)^{-1}(A'WB)E(vv')(A'WB)'(A'WA)^{-1} \quad (4)$$

or

$$P_{xv} = P_x + P_x Q P_v Q' P_x \quad (5)$$

Now  $P_{xv}$  and  $Q$  are readily formed recursively. The matrix  $P_x = (A'WA)^{-1}$  can be computed recursively, using an identity, without explicit matrix inversion if  $W^{-1}$  is diagonal (Refs. 2 or 8):

$$P_x(k+1) = P_x(k) - P_x(k)a'_{k+1} \left[ a_{k+1} P_x(k)a'_{k+1} + w_{k+1}^{-1} \right]^{-1} a_{k+1} P_x(k) \quad (6)$$

Also

$$Q(k+1) = Q(k) + a'_{k+1} w_{k+1}^{-1} b_{k+1} \quad (7)$$

where

$P_x(k), Q(k)$  signifies the evaluation of the matrices using the first  $k$  rows of  $A, B, W$

$a_{k+1}, b_{k+1}$  are the  $(k+1)^{st}$  rows of  $A, B$

$w_{k+1}^{-1}$  is the  $(k+1)^{st}$  diagonal element of  $W^{-1}$

We see that the uncertainty in  $x$ , as given by  $P_{xv}$ , is a sum of two terms, the second arising from the use of uncertain values for  $v$ . The second term can be further expanded by partitioning the matrices involved, to present separately the effects upon  $P_{xv}$  of uncertainties in individual, or groups of, parameters. Assume that  $P_v$  is block diagonal

$$P_v = \begin{bmatrix} P_{v,1} & P_{v,2} & \cdots & P_{v,s} \end{bmatrix} \quad (8)$$

and specify a compatible partition of  $B$ ,

$$B = [B_1, B_2, \cdots, B_s] \quad (9)$$

and let  $Q_i = A'WB_i$  ( $i = 1, \cdots, s$ ). Then

$$P_{xv} = P_x + P_x [Q_1 P_{v,1} Q_1' + \cdots + Q_s P_{v,s} Q_s'] P_x \quad (10)$$

and  $P_x Q_i P_{v,i} Q_i' P_x$  is the contribution to the covariance matrix  $P_{xv}$  due to the uncertainties in the  $i^{\text{th}}$  group of non-estimated parameters.

## 5. DATA EDITING AND BOUNDED ESTIMATES

Another real problem, which will have to be faced in orbital applications of MFT, is that of data editing. In the techniques used with the classical methods of orbit determination it is usually assumed that many observations are simultaneously available and that, by polynomial or other smoothing, outliers can be detected and discarded. The assumption is, however, contrary to the spirit of MFT. If observations are comparatively scarce and only available singly, the usual kind of data editing may be out of the question. There is the further possibility that an apparently bad observation is in fact valid, the large residual being due to the inadequacy of the mathematical model. In such circumstances, a practical procedure might be to reject those observations that induce an unacceptably large correction to the estimate. An alternative would be to accept all observations, but to impose a bound upon the computed correction to the estimate.

Suppose that, after recording the  $k^{\text{th}}$  observation, we seek a correction  $\Delta x(k) = x(k) - x(k-1)$  to the parameter vector that minimizes

$$f(k) = [y(k) - A(k)x(k)]' W(k) [y(k) - A(k)x(k)] \quad (11)$$

subject to the bound

$$\|G \cdot \Delta x(k)\|^2 = \sum_{i=1}^p \left( \frac{\Delta x_i}{g_i} \right)^2 = \Delta x(k)' G^2 \Delta x(k) \leq 1 \quad (12)$$

( $G$  is a diagonal matrix with elements  $g_i^{-1}$ .)

The solution of the bounded problem requires the minimization of

$$f^*(k) = f(k) + \lambda^2(k) \Delta x(k)' G^2 \Delta x(k) \quad (13)$$

where  $\lambda^2(k)$  is a positive constant to be determined. (The method is due to D. Morrison and reported in Refs. 10 and 11.) To locate the minimum of  $f^*(k)$ , we set  $\partial f^*(k)/\partial[\Delta x(k)] = 0$  and solve (with  $\lambda(k)^2 = 0$  initially) the resulting linear system

$$\begin{aligned} & [A(k)' W(k) A(k) + \lambda^2(k) G^2] \Delta x(k) \\ &= A(k)' W(k) [y(k) - A(k)x(k-1)] \\ &= [A(k-1)' W(k-1) \mid a'_k w_k] \left[ \begin{array}{c} \overline{y(k-1) - A(k-1)x(k-1)} \\ \overline{y_k - a_k x(k-1)} \end{array} \right] \end{aligned} \quad (14)$$

(the dotted bars indicate a partitioning)

$$= A(k-1)' W(k-1) [y(k-1) - A(k-1)x(k-1)] + a'_k w_k [y_k - a_k x(k-1)] \quad (15)$$

$$= a'_k w_k [y_k - a_k x(k-1)] \quad (16)$$

since the first member of Eq. (15) will be zero as the result of minimizing  $f(k-1)$ .

If the solution fails to satisfy the bounding condition, a non-zero value of  $\lambda^2(k)$  is chosen to initiate a search and iteration procedure for a value which does satisfy  $\|G \cdot \Delta x(k)\|^2 = 1$ , at least approximately. Each step requires the solution of the linear system, and, upon convergence, the solution minimizes  $f^*(k)$  subject to the bounding condition in Eq. (12).

### 5.1 APPROXIMATE INVERSION

Recall that  $P = (A'WA)^{-1}$  can be found recursively through the use of a matrix identity. By another application of the same formula, we obtain

$$[P^{-1} + (\lambda G)' I(\lambda G)]^{-1} = P - \lambda^2 P G (\lambda^2 G P G + I)^{-1} G P \quad (17)$$



in which the inverse can be approximated by truncating a series representation

$$(I + \lambda^2 GPG)^{-1} = I - \lambda^2 GPG + (\lambda^2 GPG)^2 - \dots \quad (18)$$

leading to

$$[P^{-1} + \lambda^2 G^2]^{-1} = P[I - C + C^2 - \dots] \quad (19)$$

where  $C = \lambda^2 G^2 P$ . Thus, even the bounded problem may permit a solution (although only approximate) without explicit matrix inversion.

## 5.2 SUCCESSIVE BOUNDING

Bounds may be imposed in successive stages, in which case the right member of Eq. (16) requires modification. Had the estimate  $x(k-1)$  been the result of minimizing  $f^*(k-1)$ , [that is, had  $\Delta x(k-1)$  also been the solution of an equation like Eq. (14)], then in Eq. (15) for the  $k^{\text{th}}$  stage

$$A(k-1)' W(k-1)[y(k-1) - A(k-1)x(k-1)] = \lambda^2(k-1)G^2 \Delta x(k-1) \quad (20)$$

and this quantity must be added to Eq. (16), or finally

$$[A(k)' W(k)A(k) + \lambda^2(k)G^2] \Delta x(k) = a_k' w_k [y_k - a_k x(k-1)] + \lambda^2(k-1)G^2 \Delta x(k-1) \quad (21)$$

## 6. SUMMARY AND SPECULATION

Orbit determination accuracies have improved dramatically in eight years - from the early days of estimating only orbit parameters, with simple models and empirical techniques - to today's sophisticated methods wherein realistic complex models can be used and their many parameters estimated. The emphasis has been on analysis of residuals, explanatory deterministic hypotheses, and testing by prediction. The random forcing function, common in the dynamic problems of MFT, has played no significant part in modern orbit determination of the classical type.

The drag problem, especially that of determining short-period fluctuations in density, may be a candidate for stochastic treatment. Rauch (Ref. 12) has obtained nice reductions in residuals in a smoothing application of MFT, but it is hard to foresee effective prediction from this approach. Perhaps the best use of MFT methods will be in the estimation of amplitudes, periods, and time constants as data for the formulation of deterministic hypotheses.

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## APPENDIX. EXAMPLES

All the results displayed were obtained using the program TRACE (Ref. 10), which can generate data, perform covariance analyses, and estimate trajectory, geophysical, and observational parameters. The estimation methods are basically classical and operation of the program in a filtering mode is inefficient and not feasible for extensive studies.

The reference trajectory for all the figures is a circular equatorial orbit at an altitude of 100 n mi above a spherical earth. The only perturbative force is that of atmospheric drag derived from a Jacchia model atmosphere and a ballistic coefficient  $C_D A/W = 0.1 \text{ ft}^2/\text{lb}$ . Observing stations are located at  $30^\circ$  intervals in the orbital (equatorial) plane. By specifying an interval of two minutes, with a minimum elevation of  $20^\circ$ , only one observation per pass was generated, to which independent normal random noise was added.

Two revolutions of data were fitted with both the true and a false ( $C_D A/W = 0.11$ ) model. The in-track separation between the reference and the fitted trajectories for the fitting interval, and for a like two-revolution prediction interval, is shown in Fig. 1. By using the true model, the maximum amplitudes in the reconstruction and prediction intervals are 142 and 364 ft, respectively; the curve would lie entirely within the shaded area.

In Fig. 2 the residuals for a false-model fit and prediction are displayed. Note their systematic character, the level in the reconstruction interval (10 times the random component), and the rapid growth in the prediction interval.

When the model parameters are assumed known, the covariance matrices give an in-track standard deviation of only 200 ft at the end of the prediction interval. The 10-percent change in  $C_D A/W$ , which leads to a 9-mile difference in trajectory position, causes only a 4-ft change in the in-track standard

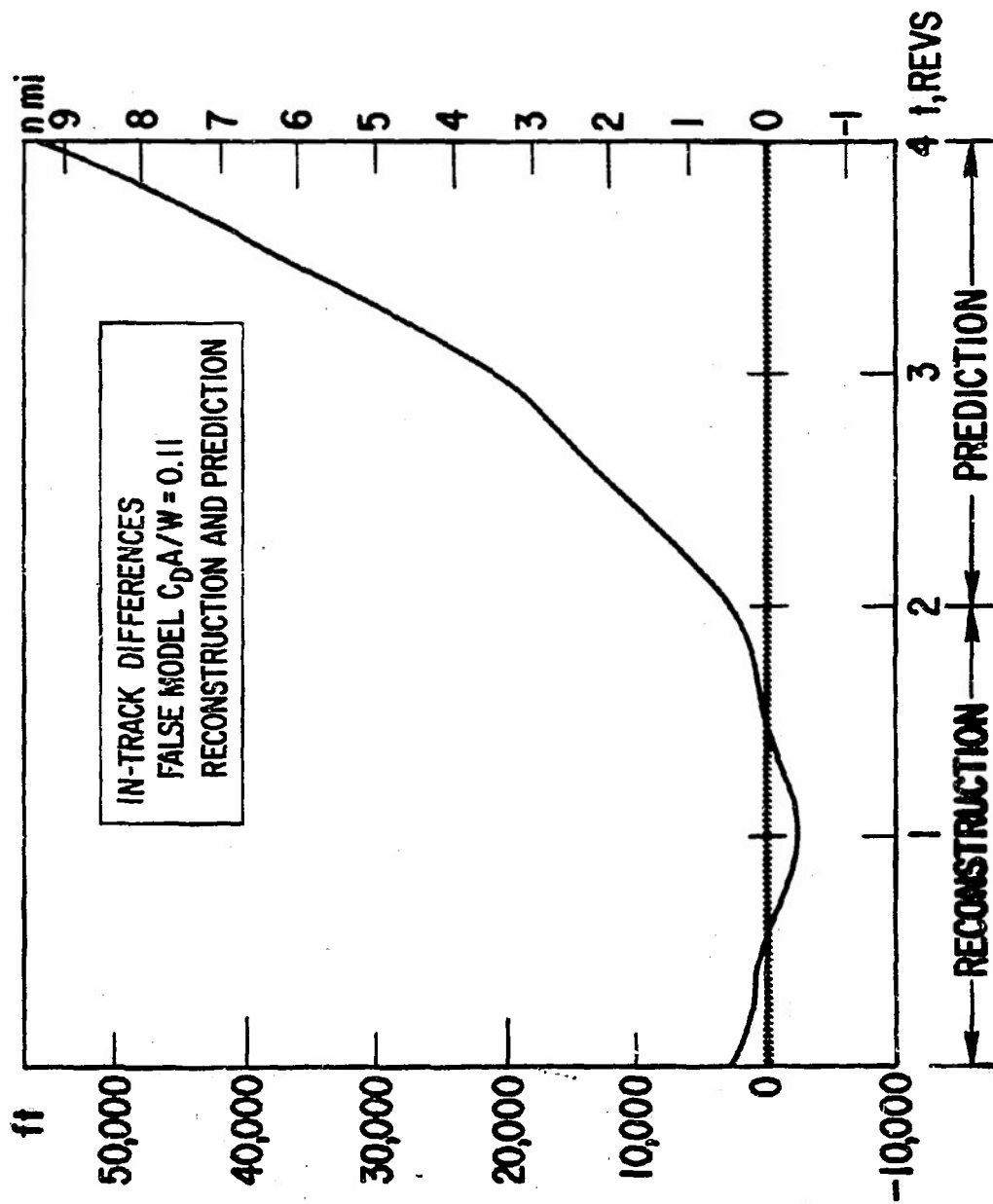


Figure 1. In-Track Differences

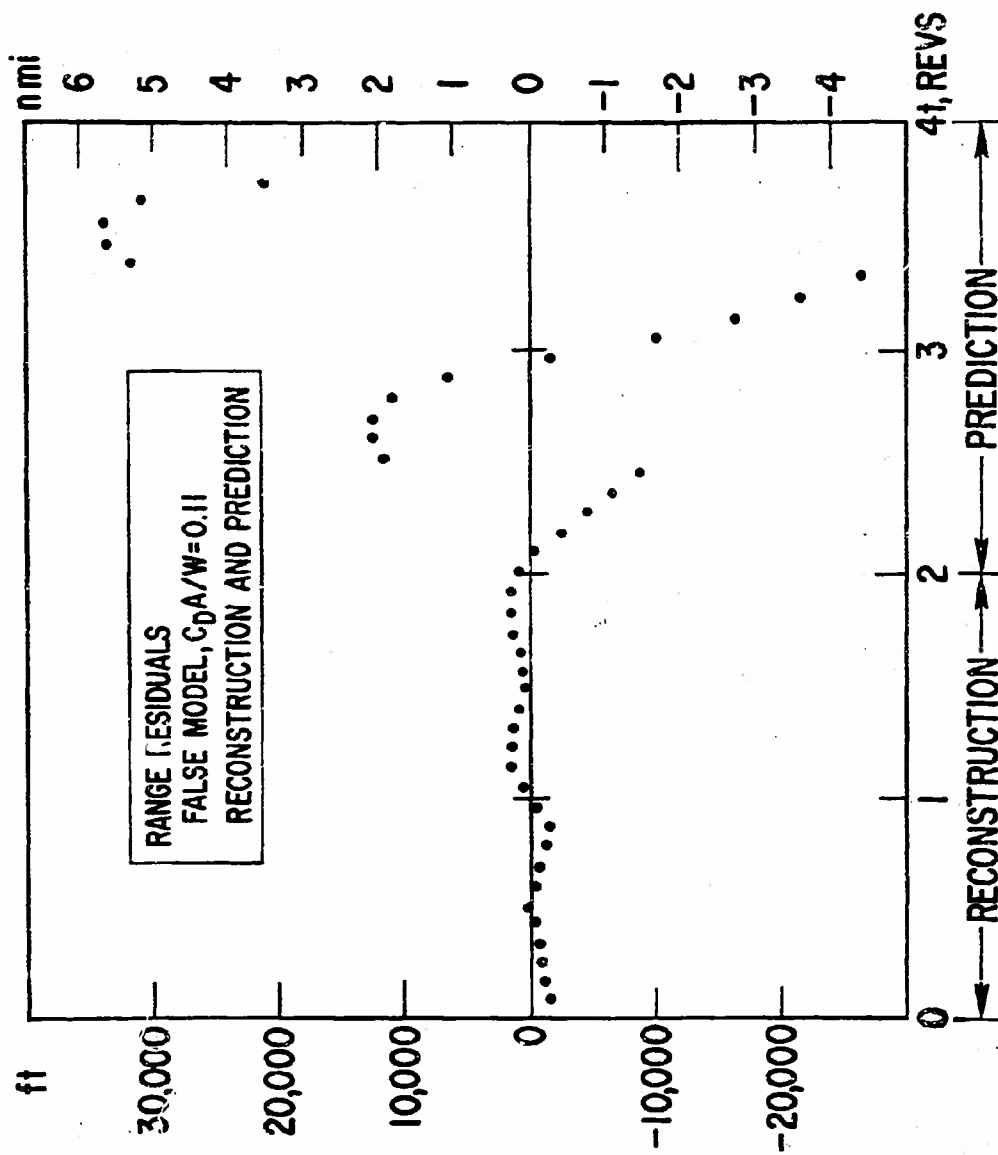


Figure 2. Range Residuals

deviation. Much more realistic statistics are produced when a 10-percent uncertainty is introduced for  $C_D A/W$ ; that is,  $P_v = 0.0001$  in Eq. (5). The results are shown in Fig. 3, in which 200 ft are represented by about half the thickness of the shaded area.

Operating TRACE in recursive modes led to the trajectory differences (from the reference trajectory) displayed in Fig. 4. The theoretical values are taken from covariance calculations and, in the prediction interval, are simply those of Fig. 3 presented in logarithmic scale. Again, the trajectory determined in the presence of a false (by 10 percent in  $C_D A/W$ ) model diverged rapidly from the reference trajectory, but the differences were accurately predicted by the  $P_{xv}$  covariance matrix.

Two runs were made in which the correct value of  $C_D A/W$  was used, and the different results illustrate the effect of the assumption that the orbit filtering problem can be linearized. In the linear mode, TRACE is mathematically a static model recursive estimation program. The initial position and velocity parameters are re-estimated with each observation, as per Eq. (16), with  $\lambda^2(k) = 0$ . In both the linear and nonlinear modes, a) the covariance matrix  $P_x$  is obtained by direction inversion rather than by appeal to the identity in Eq. (6), and b) the position of the satellite is computed from the full (nonlinear) equations of motion. In the nonlinear mode, TRACE successively fits batches of 1, 2, 3 . . . observations without iteration. The matrix A and the residuals y are recomputed for each fit and thus the nonlinearities in the covariance matrix and residual vector are taken into account. Standard deviations of 20,000 ft and 40 fps were used in constructing the initial covariance matrix required for recursive estimation.



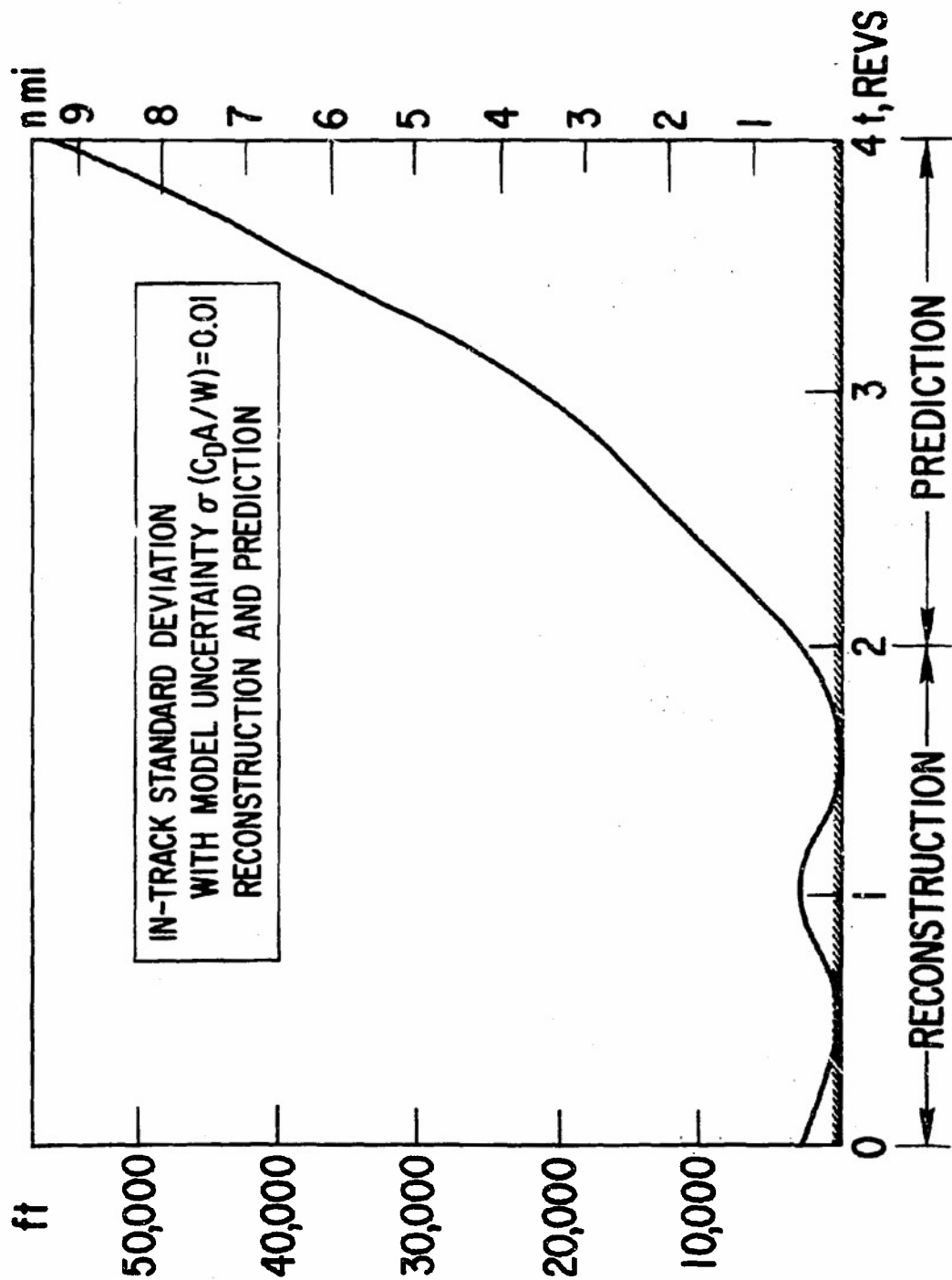


Figure 3. In-Track Standard Deviation

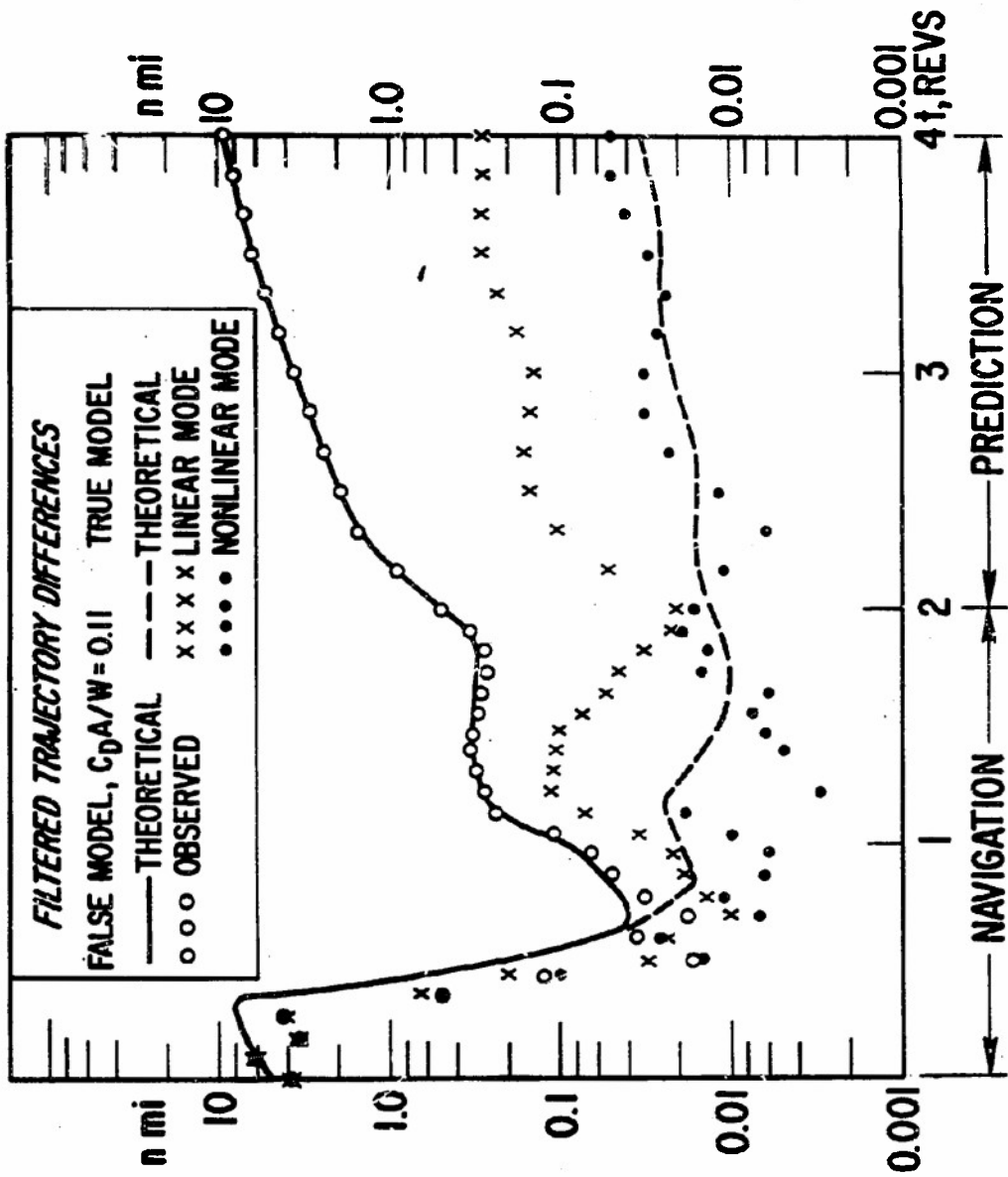


Figure 4. Filtered Trajectory Differences

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